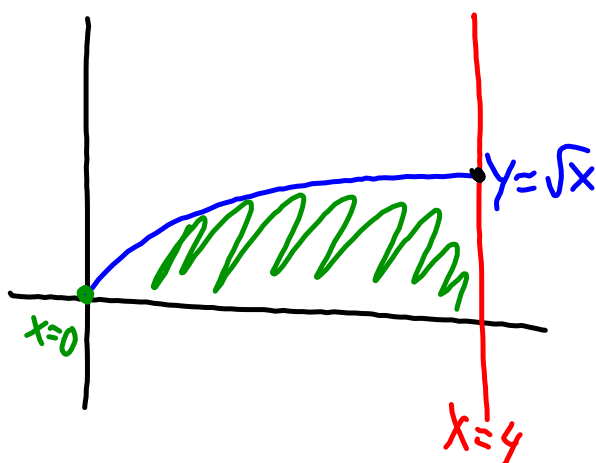


5/19/17 "There is no traffic on the extra mile."  
- Anonymous

Final on Wednesday 6/7

AIM: How do we find volume?

1) Find the area of the region "R" bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 4$ .



$$\text{Area} = \int_0^4 (\sqrt{x} - 0) dx \quad \text{Top - bottom}$$

$$= \int_0^4 \sqrt{x} \, dx$$

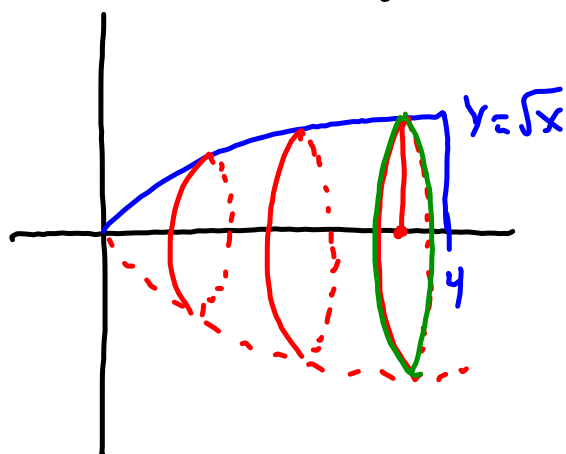
$$= \int_0^4 x^{1/2} \, dx$$

$$= \left[ \frac{2}{3} x^{3/2} \right]_0^4$$

$$= \frac{2}{3} (4)^{3/2} - \frac{2}{3} (0)^{3/2}$$

$$= \boxed{\frac{16}{3}}$$

2) If we rotate "R" about the x-axis. Find the volume of the resulting solid.



each slice is a circle

$$\text{Area of circle} = \pi r^2$$

$$r = \sqrt{x}$$

$$\begin{aligned} \text{area of each cross section} \\ = \pi (\sqrt{x})^2 \end{aligned}$$

$$\text{Volume} = \int \text{Cross sectional area.}$$

$$\text{Volume} = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$\begin{aligned} \text{Volume} &= \pi \int_0^4 (\sqrt{x})^2 dx \\ &= \pi (8) \end{aligned}$$

$$V = 8\pi \text{ units}^3$$

$$f(x) = 7x^6$$

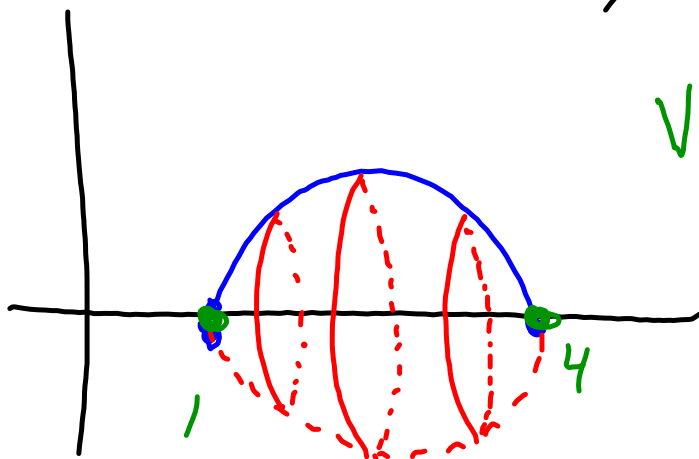
$$f'(x) = 42x^5$$

$$\begin{array}{c} y = 7x^6 \\ \swarrow \quad \searrow \\ 7 \cdot 6x^5 \\ 42x^5 \end{array}$$

3)  $f(x) = -x^2 + 5x - 4$

Let  $R$  be the region in the 1<sup>st</sup> quadrant between  $f(x)$  and the x-axis.

What is the Volume of the solid formed by rotating " $R$ " about the x-axis?



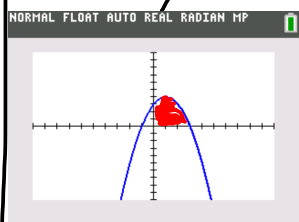
$$V = \pi \int_a^b (f(x))^2 dx$$

$$V = \pi \int_1^4 (-x^2 + 5x - 4)^2 dx$$

$$V = \pi (8.1)$$

$$V = 8.1\pi \text{ units}^3$$

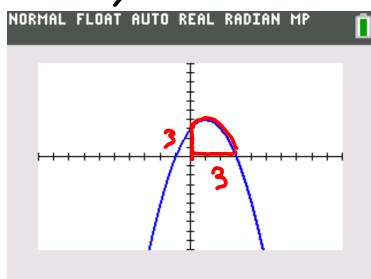
- 4) Let  $R$  be the region in  $Q1$  bounded by  $y = 3 + 2x - x^2$ ,  $y = 0$  and  $x = 0$
- a) Find the Area of  $R$ .



$$\text{Area} = \int_0^3 (3 + 2x - x^2 - 0) dx$$

$$\text{Area} = 9 \text{ units}^2$$

- b) Find the Perimeter of " $R$ "



$$P = 3 + 3 + \int_0^3 \sqrt{1 + (y')^2} dx$$

$$P = 3 + 3 + \int_0^3 \sqrt{1 + (2 - 2x)^2} dx$$

$$y = 3 + 2x - x^2$$

$$y' = 2 - 2x$$

$$3 + 3 + \int_0^3 (\sqrt{1 + (2 - 2x)^2}) dx$$

12.12572662

$$P = 12.126 \text{ units}$$

- c) Find the volume of the solid generated by rotating  $R$  about the  $x$ -axis.

$$V = \pi \int_0^3 (3 + 2x - x^2)^2 dx$$

$$\int_0^3 ((3 + 2x - x^2)^2) dx$$

30.6

$$V = 30.6 \pi \text{ units}^3$$

